**Mathematical Logic**

**1. Connectives**

Logical connectives are symbols used to connect statements in logic. The basic connectives are:

- **Negation (¬)**: If *P* is a statement, then ¬P (not P) is its negation.
- **Conjunction (∧)**: $P \wedge Q$ is true if both *P* and *Q* are true.
- **Disjunction (∨)**: $P \vee Q$ is true if at least one of *P* or *Q* is true.
- **Implication (→)**: $P \rightarrow Q$ means if *P* is true, then *Q* must be true.
- **Biconditional (↔)**: $P \leftrightarrow Q$ means *P* and *Q* are both true or both false.

**2. Truth Tables**

A truth table is a table that shows all possible truth values of logical expressions.

Example for basic connectives:

| P | Q | ¬P | P ∧ Q | P ∨ Q | P → Q | P ↔ Q |
|---|---|---|---|---|---|---|
| T | T | F | T | T | T | T |
| T | F | F | F | T | F | F |
| F | T | T | F | T | T | F |
| F | F | T | F | F | T | T |

**3. Tautologies and Contradictions**

- **Tautology**: A logical expression that is always true (e.g., $P \vee \neg P$).
- **Contradiction**: A logical expression that is always false (e.g., $P \wedge \neg P$).
- **Contingency**: A logical expression that is neither always true nor always false.

**4. Equivalence and Implications**

- Two statements *P* and *Q* are **logically equivalent** if $P \leftrightarrow Q$ is always true.
- **Implication Rules:** If $P \rightarrow Q$ is always true, *P* is said to **imply** *Q*.

**5. NAND and NOR Gates**

- **NAND (Not AND):** $(P \wedge Q)'$
- **NOR (Not OR):** $(P \vee Q)'$

- **Universal Gates:** NAND and NOR are universal gates, meaning any logical function can be implemented using only these gates.

**6. Normal Forms (CNF & DNF)**

- **Conjunctive Normal Form (CNF):** A logical expression written as a conjunction of disjunctions.
    - Example: $(P \vee Q) \wedge (\neg P \vee R)$
- **Disjunctive Normal Form (DNF):** A logical expression written as a disjunction of conjunctions.
    - Example: $(P \wedge Q) \vee (\neg P \wedge R)$

**7. Converting Expressions to CNF and DNF**

Steps to convert to CNF:

1. Eliminate implications using $P \rightarrow Q \equiv \neg P \vee Q$
2. Move negations inside using De Morgan's laws.
3. Convert into conjunction of disjunctions.

Steps to convert to DNF:

1. Use the same steps as CNF but convert into disjunction of conjunctions.

**8. Theory of Inference**

- **Modus Ponens**: If $P \rightarrow Q$ and $P$ is true, then $Q$ must be true.
- **Modus Tollens**: If $P \rightarrow Q$ and $Q$ is false, then $P$ must be false.
- **Hypothetical Syllogism**: If $P \rightarrow Q$ and $Q \rightarrow R$, then $P \rightarrow R$.
- **Disjunctive Syllogism**: If $P \vee Q$ and ¬P, then $Q$ must be true.

**9. Propositional Calculus**

- Deals with propositions and their logical relationships.
- Includes axioms and rules of inference to derive new statements.

**10. Predicate Calculus (Introduction)**

- **Propositional logic** deals with whole statements.
- **Predicate logic** introduces variables, functions, and quantifiers.

**11. Predicates and Quantifiers**

- **Predicate**: A statement containing variables (e.g., $P(x): x > 5$).

- **Quantifiers**:
    - **Universal Quantifier (∀)**: ∀x P(x) means P(x) is true for all x.
    - **Existential Quantifier (∃)**: ∃x P(x) means P(x) is true for at least one x.