Graph Theory Notes

1. Basic Definition of a Graph

A **graph** is a mathematical structure used to model pairwise relationships between objects. A graph consists of:

- A **set of vertices (nodes)**, denoted as
- A **set of edges (links)**, denoted as
- An edge connects two vertices and , written as

Graph Representation

Diagram:

(A)-----(B)

| |

| |

(C)-----(D)

2. Types of Graphs

Directed Graph (Digraph)

- A graph where edges have a direction.
- Each edge is represented as an ordered pair meaning there is a directed edge from vertex to .

Diagram:

A → B

↓ ↓

C → D

Undirected Graph

- A graph where edges have no direction.
- An edge between and is denoted as and is bidirectional.

Diagram:

A --- B

| |

C --- D

Weighted Graph

- A graph where edges have associated weights (numerical values representing costs, distances, etc.).

Diagram:

A --5-- B

| |

3| |2

C --1-- D

3. Representation of Graphs in Computers

Adjacency Matrix

- A matrix where:
    - if there is an edge from vertex to vertex .
    - otherwise.
- Space Complexity:
- Efficient for dense graphs.

Example:

A B C D

A [0 1 1 0]

B [1 0 0 1]

C [1 0 0 1]

D [0 1 1 0]

Adjacency List

- Each vertex has a list of adjacent vertices.
- Stored as an array or a dictionary.
- Space Complexity:
- Efficient for sparse graphs.

Example:

A: B, C

B: A, D

C: A, D

D: B, C

4. Degree of Vertices

Indegree and Outdegree

- **Indegree** of a vertex: Number of incoming edges.
- **Outdegree** of a vertex: Number of outgoing edges.

5. Paths, Cycles, and Acyclic Graphs

- **Path**: A sequence of edges connecting vertices.
- **Cycle**: A path where the first and last vertex are the same.
- **Acyclic Graph**: A graph without cycles.

6. Simple Operations on Graphs

Common Graph Operations and Their Computation Costs

Operation Adjacency Matrix Adjacency List

Add Edge

Remove Edge

Check Edge Existence

Find Neighbors

7. Connected Graph, Tree, and Forest

- **Connected Graph**: A graph where there is a path between every pair of vertices.
- **Tree**: A connected acyclic graph.
- **Forest**: A collection of disjoint trees.

Tree Diagram:

/\ \

D E F

8. Bipartite Graph

- A graph whose vertices can be divided into two sets such that no two adjacent vertices belong to the same set.
- Can be checked using BFS or DFS.

Diagram:

Set1: {A, C, E}

Set2: {B, D, F}

A---B

| |

C---D

| |

E---F

9. Graph Traversal Algorithms

Breadth-First Search (BFS)

- Uses a queue (FIFO structure).
- Visits all neighbors of a vertex before moving deeper.
- **Time Complexity**:

BFS Example:

Graph: A - B - C - D

Start at A → B → C → D

Depth-First Search (DFS)

- Uses a stack (or recursion).
- Explores as far as possible before backtracking.
- **Time Complexity**:

DFS Example:

Graph: A - B - C - D

Start at A → C → D → B

These enhanced notes include diagrams and explanations for better understanding of Graph Theory.